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Abstract

We consider a principal-agent relationship with adverse selection. Principals pay informational rents due to asymmetric information and sell their output in a homogeneous Cournot-oligopoly. We find that asymmetric information may mitigate or more than compensate the welfare reducing impact of market power, irrespective of whether the number of firms is given exogenously or determined endogenously by a profit constraint. We further show that welfare in a setting with adverse selection may be higher than the maximized welfare level attainable in a world with perfect observability.

Keywords: Adverse Selection, Oligopoly, Welfare

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1 Introduction

The rent-efficiency trade-off implies a welfare loss in a principal-agent relationship with adverse selection. Usually, this result is derived for simple bilateral negotiations between, for instance, a firm or its owner (principal) and one manager (agent) who features either high or low costs of production. Moreover, there generally is no interaction on the output market.[1] However, pure monopolies, perfect competition or a constant demand per firm are rare market characteristics. Instead, "(o)ligopoly is pervasive in our daily live" (Head and Spencer, 2017, p. 1415).

In this paper, we study an adverse selection problem in an oligopolistic market. We show that the characteristics of the output market can fundamentally affect the rent-efficiency trade-off. As a consequence, expected welfare in a Cournot-oligopoly with asymmetric information can be higher than the level of expected welfare resulting in a setting with perfect observability in which firms know the type of their manager at the contracting stage. This outcome may even be observed for the socially optimal number of firms and in a setting in which the number of competitors is determined endogenously by a profit constraint.

Our main finding can be explained by two countervailing effects. On the one hand, expected aggregate output is lower in case of asymmetric information than in a world with perfect observability, such that expected welfare, c.p., decreases. On the other hand, the variance in quantities is higher for asymmetric information than for perfect observability, which, c.p., increases expected welfare. The variance effect arises because output of the low-cost manager increases in an oligopolistic market with asymmetric information, which complements the standard finding that output of the high-cost manager is reduced. Hence, there are distortions at the top and at the bottom. If the number of firms increases, the variance effect rises by more in the asymmetric information setting than under perfect observability. Accordingly, if the number of competitors exceeds a threshold, the welfare-enhancing effect of a higher variance dominates the welfare-reducing effect of lower expected aggregate output, and expected welfare under asymmetric information is higher than for perfect observability.

[1] Seminal papers are Stiglitz (1977), Guesnerie and Laffont (1984) and Demski and Sappington (1984).

Our analysis is primarily related to studies that investigate adverse selection in oligopolistic markets.[2] Piccolo (2011) focuses on entry decisions in a Salop (1979) type model with asymmetric information and derives conditions under which greater uncertainty makes entry more costly. Etro and Cella (2013) investigate the effects of competitors on incentive schemes and R&D investment decisions. Bonazzi et al. (2021) analyze manufacturer-retailer hierarchies and compare the welfare consequences of different vertical price restraints under asymmetric information.[3] Furthermore, given that there is ex-ante uncertainty about productivity, our analysis features mechanisms, which are also present in investigations of the effects of price (or cost) uncertainty in the tradition of Waugh (1944), Oi (1961), and Massell (1969).

The relationship between welfare and the number of firms is also relevant in analyses of excessive entry in Cournot-oligopolies (see von Weizsäcker, 1980, Mankiw and Whinston, 1986, and Suzumura and Kiyono, 1987). The robustness of this outcome has been evaluated from a variety of perspectives. Our analysis is particularly related to investigations, which consider excessive entry in the presence of uncertainty. Creane (2007) assumes that firms incur sunk costs but take up production with a probability, which is strictly less than one. Deo and Corbett (2009) and Jansen and Özaltın (2018) consider settings in which output is stochastic, for a given level of inputs, while de Pinto and Goerke (2022) focus on cost uncertainty. In contrast to our analysis, none of the contributions compares the welfare effects of different informational structures.[4]

Finally, analyses in the tradition of Hirshleifer (1971) show that the welfare effects of being able to acquire information may not be positive (see also, for example, Marshall, 1974, Eckwert and Zilcha, 2003, de Garidel-Thoron, 2005 and Maurer and Tran, 2018).[5] The so-called *Hirshleifer-effect* can arise because additional information about the state of the economy impedes risk-sharing through trade. In this paper, however, we focus on the availability of information about firm-specific productivity, which is ex-ante uncertain.

[2]Perfectly competitive markets have been studied, for instance, by Hart (1983), Scharfstein (1988) and more recently by Pouyet et al. (2008) and Guerrieri et al. (2010).

[3]There are also some studies that scrutinize the implications of adverse selection in duopoly and, consequently, cannot consider the effects of the number of firms, which play a decisive role in our analysis (see, e.g., Martimort, 1996, Piccolo and Pagnozzi, 2013).

[4]Mukherjee and Tsai (2014) consider the excessive entry prediction in the presence of managerial delegation, but neither incorporate adverse selection nor consider the welfare impact of delegation.

[5]We are grateful to an anonymous referee who drew our attention to this line of enquiry.

In the subsequent Section 2, we outline the model and characterize market outcomes for both informational settings, namely a situation with perfect observability of the manager's type and a framework with asymmetric information. In Section 3, we compare the resulting levels of expected welfare and derive our main result for an exogenously given number of firms. We extend the model and allow for firm entry in Section 4, while Section 5 concludes. We relegate proofs, some more elaborate derivations, and the formal analysis of the extension considered in Section 4 to the Appendix.

2 The Model

2.1 Set-up

In our basic framework, an exogenously given number of n firms compete in a Cournot-oligopoly. Each firm is owned by a principal. Lacking in managerial expertise, production is only feasible if the owner delegates governance of the firm to an agent, whom we refer to as manager. We index owners and managers, who are both risk-neutral, by $i = 1, 2, ..., n$. Each unit of effort, which a manager provides, generates one unit of output, q.

To facilitate closed-form solutions, the (inverse) demand schedule is linear, $P(Q) = a - bQ$, with P being the price, a the choke price, and b a positive parameter. Total output is denoted by Q and consumer surplus is given by $CS = bQ^2/2$. A manager's outside options is, for simplicity, normalized to zero. As a measure of expected welfare, we employ the sum of expected profits of all n firms, expected consumer surplus, and expected utility of all n managers.

Owners incur sunk costs, $F > 0$, to settle production. The existence of sunk costs is a necessary condition for expected welfare to be an inverted U-shaped function of the number of firms, and, therefore, ensures an interior maximizer. Consequently, a benevolent social planner would establish an oligopoly. In addition, sunk costs guarantee a finite market concentration if the number of firms is determined endogenously in market equilibrium by a profit constraint. Hence, sunk costs provide a micro-foundation for the market structure we study.

A manager's utility equals the wage, w, minus the costs of production, θq, with $\theta \in$

$\Theta = \{\underline{\theta}, \overline{\theta}\}$. The probability that a manager has low marginal costs is $Pr[\theta = \underline{\theta}] = v$ and the converse probability is $Pr[\theta = \overline{\theta}] = 1 - v$, $\overline{\theta} > \underline{\theta}$, $0 < v < 1$. We assume that types are uncorrelated and we denote the spread between marginal costs by $\Delta\theta \triangleq \overline{\theta} - \underline{\theta}$. Therefore, expected unit production costs equal $\mathbb{E}[\theta] = v\underline{\theta} + (1 - v)\overline{\theta}$. We refer to a menu of contracts as $\{w_i(\theta), q_i(\theta)\}_{\theta \in \Theta}$ and, to simplify notation, use $\underline{q}_i \triangleq q_i(\underline{\theta})$, $\overline{q}_i \triangleq q_i(\overline{\theta})$ as well as $\underline{w}_i \triangleq w_i(\underline{\theta})$, and $\overline{w}_i \triangleq w_i(\overline{\theta})$. For each owner and a θ-type manager, in realization, profits π_i and utility U_i are:

$$\pi_i(w_i(\theta), q_i(\theta), Q) = (a - bQ)q_i(\theta) - w_i(\theta) - F, \tag{1}$$

$$U_i(w_i(\theta), q_i(\theta), \theta) = w_i(\theta) - \theta q_i(\theta). \tag{2}$$

The timing is as follows:

1. Managers realize their type.
2. Each owner meets one manager and offers a contract, i.e., determines the incentive scheme.
3. If the manager accepts, the contract is executed. Otherwise, the manager receives the outside option of zero and the owner incurs a loss of F.
4. All firms that have hired a manager compete in quantities in the product market.
5. Payments are made and profits are realized.

We consider two informational structures. The first presumes that the owner can perfectly observe the own manager's type at the contracting stage and, hence, conditions the contract offered on the manager's productivity. We use the subscript p to indicate this scenario. In the second setting with asymmetric information, denoted by the subscript a, marginal production costs are the manager's private knowledge.[6]

For both informational structures, we study a Bayesian Cournot-Nash equilibrium, in which each firm i has a belief about the aggregate output produced by all other firms, Q_{-i}. Given that types are uncorrelated, knowledge of the own manager's type does not provide a firm with information about marginal costs of other firms. From firm i's point

[6] Given the sequence of decisions, assuming risk-averse managers would not affect incentives since they know their type at the contracting stage. However, risk attitudes by owners change market fundamentals. Nonetheless, our results are basically unaffected by allowing for risk-aversion. A proof of this claim is available from the authors upon request.

of view, in expectation $(n-1)v$ firms are matched with a $\underline{\theta}$-type manager, while the remaining $(n-1)(1-v)$ firms employ a $\overline{\theta}$-type manager.

Focusing on a symmetric equilibrium, in which firms of the same type offer identical contracts, firm i's expected profits and the expected output of all other firms are given by:

$$\mathbb{E}[\pi_i] = v[(a - b\underline{q}_i - b\mathbb{E}[Q_{-i}])\underline{q}_i - \underline{w}_i] + (1-v)[(a - b\overline{q}_i - b\mathbb{E}[Q_{-i}])\overline{q}_i - \overline{w}_i] - F, \tag{3}$$

$$\mathbb{E}[Q_{-i}] = \sum_{j \neq i} \mathbb{E}[q_j(\theta)] = (n-1)[v\underline{q} + (1-v)\overline{q}], \tag{4}$$

respectively, where all terms in expectations are computed with respect to the distribution of marginal costs, θ. Since firms are (ex-ante) identical, we subsequently omit the index i. Moreover, we assume n to be a continuous variable.

Each firm stochastically produces $q(\theta)$. Thus, expected aggregate output is the sum of n i.i.d. random variables, such that $\mathbb{E}[Q] = n\mathbb{E}[q(\theta)]$ and $\text{Var}[Q] = n\text{Var}[q(\theta)]$. Expected welfare $\mathbb{E}[W]$ can be expressed as:

$$\mathbb{E}[W] = \mathbb{E}[(a - \frac{b}{2}Q)Q - n\theta q(\theta) - nF]. \tag{5}$$

2.2 Perfect Observability

Under perfect observability, an owner observes the type of the manager matched with before a contract is offered and maximizes (3), subject to the participation constraint

$$U\left(w(\theta), q(\theta), \theta\right) = w(\theta) - \theta q(\theta) \geqslant 0 \qquad \forall\, \theta \in \left\{\underline{\theta}, \overline{\theta}\right\}. \qquad PC(\theta)$$

The wage compensates for the costs of production and leaves the manager with the zero reservation utility. The output choices equalize marginal revenues with marginal costs. This yields:

Lemma 1. *Under perfect observability, there exists a unique equilibrium in which quan-*

tities, expected aggregate output, expected price and expected profits are:

$$\underline{q}_p = \frac{a-\underline{\theta}}{b(1+n)} + \frac{(n-1)(1-v)}{2b(1+n)}\Delta\theta, \tag{6}$$

$$\overline{q}_p = \frac{a-\overline{\theta}}{b(1+n)} - \frac{(n-1)v}{2b(1+n)}\Delta\theta, \tag{7}$$

$$\mathbb{E}[Q_p] = \frac{n(a-\mathbb{E}[\theta])}{b(1+n)}, \tag{8}$$

$$\mathbb{E}[P_p] = \frac{a+n\mathbb{E}[\theta]}{1+n}, \tag{9}$$

$$\mathbb{E}[\pi_p] = \frac{(a-\mathbb{E}[\theta])^2}{b(1+n)^2} + \underbrace{\frac{v(1-v)\Delta\theta^2}{4b}}_{\frac{b}{n}\mathrm{Var}[Q_p(n)]} - F. \tag{10}$$

Proof Lemma 1.

See Appendix A.1.

This equilibrium has the following properties. First, expected profits increase in the variance of output due to the convexity of the profit function w.r.t. price.[7] Second, the quantity produced by a low-cost manager is higher than the respective value of a high-cost manager, i.e., $\underline{q}_p(n) > \overline{q}_p(n)$. Third, quantities, expected profits, and the expected price decline in the number of firms, while the expected aggregate output increases in n.

While we allow for monopoly as limiting case to compare our findings to those of earlier contributions, we focus our formal analysis on an oligopoly in which all firms produce positive levels of output. Therefore, we subsequently impose three restrictions:

R_p^1 The number of firms exceeds one ($n > 1$), which requires $\mathbb{E}[\pi_p(n=1)] > 0$.

R_p^2 There is an upper bound for the number of firms, denoted by n_p^e, where expected profits are equal to zero, i.e., $\mathbb{E}[\pi_p(n_p^e)] = 0$.

R_p^3 All firms in the feasible range $(1, n_p^e]$ produce positive quantities, i.e., firms offer contracts to both types of managers, and there is no shut-down of firms employing high-cost managers. Formally, this requires $\overline{q}_p(n) > 0 \ \forall\ n \in (1, n_p^e]$.

[7]See, for instance, Bergstrom and Varian (1985) in Cournot settings and Waugh (1944), Oi (1961), or Massell (1969) for the basic idea focusing on price variability.

R_p^3 together with R_p^1, R_p^2, and (10) imply that all firms realize non-negative expected profits, i.e., $\mathbb{E}[\pi_p(n)] \geqslant 0 \, \forall \, n \in (1, n_p^e]$. To ensure that R_p^1 and R_p^3 are fulfilled, we introduce two conditions (P_1) and (P_2) that are related to the sunk costs, F. First, $\mathbb{E}[\pi_p(n=1)] > 0$ implies that

$$F < \frac{1}{4b}\left[(a - \mathbb{E}[\theta])^2 + v(1-v)\Delta\theta^2\right]. \tag{P_1}$$

Second, a sufficient condition for $\overline{q}_p(n) > 0 \; \forall \; n \in (1, n_p^e]$ is $\overline{q}_p(n_p^e) > 0$. Given

$$n_p^e = \frac{a - \mathbb{E}[\theta]}{\sqrt{b\left(F - \frac{v(1-v)\Delta\theta^2}{4b}\right)}} - 1, \tag{11}$$

from (10), and inserting n_p^e in (7), we can solve $\overline{q}_p(n_p^e) > 0$ to state as equivalent condition (see Appendix A.1):

$$F > \frac{v\Delta\theta^2}{4b}. \tag{P_2}$$

Inspection of (P_1) and (P_2) shows that both conditions are not conflicting and can be fulfilled simultaneously.

2.3 Asymmetric Information

In the presence of asymmetric information, a manager's type is private information when the owner offers a contract. Each owner defines a mechanism $\langle q(\theta), w(\theta)\rangle$, which entails a transfer w for the observable and verifiable output. By the revelation principle, we analyze a direct revelation mechanism and managers truthfully reveal their types.

Formally, owners maximize expected profits, as defined in (3), by choosing wages and quantities, subject to the participation constraints of managers, $PC(\theta)$, and the incentive compatibility constraints

$$U\left(\underline{w}, \underline{q}, \underline{\theta}\right) \geqslant U\left(\overline{w}, \overline{q}, \underline{\theta}\right), \tag{$IC\left(\underline{\theta}\right)$}$$

$$U\left(\overline{w}, \overline{q}, \overline{\theta}\right) \geqslant U\left(\underline{q}, \underline{w}, \overline{\theta}\right). \tag{$IC\left(\overline{\theta}\right)$}$$

As it is standard (see e.g., Laffont and Martimort, 2002, and Appendix A.2), the participation constraint of the low-cost type, $\underline{\theta}$, is satisfied and the incentive constraint of the high-cost type, $\overline{\theta}$, is slack at the optimum. Moreover, the other two constraints bind at the optimum. Therefore, an owner's problem reduces to:

$$\underset{\{\underline{q},\overline{q}\}}{Max}\,\mathbb{E}\left[\pi_a\right] = v((a - b\underline{q} - b\mathbb{E}[Q_{-i}] - \underline{\theta})\underline{q} + (1-v)\left(a - b\overline{q} - b\mathbb{E}[Q_{-i}] - \overline{\theta}\right)\overline{q} - v\Delta\theta\overline{q} - F. \quad (12)$$

Making use of the first-order conditions and the binding constraints, contracts can implicitly be defined as:

$$\begin{array}{ll} a - b\mathbb{E}[Q_{-i}] - 2b\underline{q}_a = \underline{\theta} & a - b\mathbb{E}[Q_{-i}] - 2b\overline{q}_a = \overline{\theta} + \frac{v}{1-v}\Delta\theta \\ \underline{w}_a = \underline{\theta}\underline{q}_a + \Delta\theta\overline{q}_a & \overline{w}_a = \overline{\theta}\overline{q}_a \end{array}. \quad (13)$$

Accordingly, each low-cost type obtains a rent, $\Delta\theta\overline{q}_a$, and each firm pays an expected rent, which is given by $\mathbb{E}[rent_a] = v\Delta\theta\overline{q}_a$. This yields:

Lemma 2. *Under asymmetric information, there exists a unique equilibrium, which is separating. Quantities, expected aggregate output, expected price and expected profits are:*

$$\underline{q}_a = \frac{a - \underline{\theta}}{b(1+n)} + \frac{n-1}{2b(1+n)}\Delta\theta, \quad (14)$$

$$\overline{q}_a = \frac{a - \overline{\theta}}{b(1+n)} - \frac{1}{2b}\frac{v}{1-v}\Delta\theta, \quad (15)$$

$$\mathbb{E}[Q_a] = \frac{n(a - \overline{\theta})}{b(1+n)}, \quad (16)$$

$$\mathbb{E}[P_a] = \frac{a + n\overline{\theta}}{1+n}, \quad (17)$$

$$\mathbb{E}[\pi_a] = \frac{(a - \overline{\theta})^2}{b(1+n)^2} + \underbrace{\frac{v\Delta\theta^2}{4b(1-v)}}_{\frac{b}{n}\mathrm{Var}[Q_a(n)]} - F. \quad (18)$$

Proof Lemma 2.
See Appendix A.2.

This equilibrium has similar properties as the one under perfect observability. In particular, $\underline{q}_a(n) > \overline{q}_a(n)$ holds, showing that the monotonicity constraint is fulfilled and that the equilibrium in case of asymmetric information is always separating. Moreover, expected profits increase in the variance of output, quantities, expected profits, and the expected price decline in the number of firms, while the expected aggregate output rises with n.

As before, we impose three restrictions:

R_a^1 The number of firms is strictly greater than one, which requires $\mathbb{E}[\pi_a(n=1)] > 0$.

R_a^2 There is an upper bound for the number of firms, denoted by n_a^e, and defined by $\mathbb{E}[\pi_a(n_a^e)] = 0$.

R_a^3 There is no shut-down of firms employing high-cost managers, i.e., $\overline{q}_a(n) > 0 \ \forall \ n \in (1, n_a^e]$.

R_a^3 together with R_a^1, R_a^2, and (18) imply that all firms realize non-negative expected profits, i.e., $\mathbb{E}[\pi_a(n)] \geqslant 0 \ \forall \ n \in (1, n_a^e]$. To ensure $\mathbb{E}[\pi_a(n=1)] > 0 \ \forall \ n \in (1, n_p^e]$, we assume:

$$F < \frac{1}{4b}\left[(a-\overline{\theta})^2 + \frac{v}{1-v}\Delta\theta^2\right]. \tag{A_1}$$

A sufficient condition for $\overline{q}_a(n) > 0 \ \forall \ n \in (1, n_a^e]$ is $\overline{q}_a(n_a^e) > 0$. Given

$$n_a^e = \frac{a-\overline{\theta}}{\sqrt{b\left(F - \frac{v\Delta\theta^2}{4b(1-v)}\right)}} - 1, \tag{19}$$

from (18), and inserting n_a^e in (15), we can solve $\overline{q}_a(n_a^e) > 0$ to establish as equivalent condition (see Appendix A.2):

$$F > \frac{v\Delta\theta^2}{4b(1-v)^2}. \tag{A_2}$$

Inspection of (A_1) and (A_2) shows that both conditions are not conflicting and can be fulfilled simultaneously.

Moreover, a comparison of (6) and (7) with (14) and (15) clarifies that $\underline{q}_a(n) > \underline{q}_p(n) > \overline{q}_p(n) > \overline{q}_a(n)$ for $n > 1$. This implies that $\overline{q}_a(n) > 0$ is a sufficient condition to ensure that all quantities are positive, irrespective of the informational structure. Put differently, if condition (A_2) is fulfilled, (P_2) holds as well.

The subsequent results are particularly noteworthy when comparing the outcomes of the two informational structures as summarized in Lemmas 1 and 2:
First, expected aggregate output under perfect observability is higher than in the case of asymmetric information ($\mathbb{E}[Q_p] > \mathbb{E}[Q_a]$).

Second, the variance of aggregate output is higher if information is distributed asymmetrically ($\text{Var}[Q_a(n)] > \text{Var}[Q_p(n)]$).

Third, the quantities in a setting with asymmetric information decrease by less with a given increase in the number of firms than in a world with perfect observability ($\frac{\partial \underline{q}_p}{\partial n} = \frac{\partial \overline{q}_p}{\partial n} < \frac{\partial \underline{q}_a}{\partial n} = \frac{\partial \overline{q}_a}{\partial n} < 0$; see Appendix A.3). These differential variations in quantities come about because the reduction in output, due to the increase in the number of firms, lowers the informational rent in a setting with asymmetric information, which depends on the quantity produced by the high-cost type.[8] Hence, a less severe rent-efficiency trade-off mitigates the output reduction, relative to a world with perfect observability.

Fourth, since the expected quantity per firm under perfect observability is higher than in a world asymmetric information, the expected aggregate quantity rises by more with the number of competitors in the former than the later informational setting, such that the decrease in the price is more pronounced ($\frac{\partial \mathbb{E}[Q_p]}{\partial n} > \frac{\partial \mathbb{E}[Q_a]}{\partial n} > 0$; $\frac{\partial \mathbb{E}[P_p]}{\partial n} < \frac{\partial \mathbb{E}[P_a]}{\partial n} < 0$; see Appendix A.3)).

Fifth, expected profit in a world with asymmetric information decrease by less with the number of firms than expected profits in a setting with perfect observability ($\frac{\partial \mathbb{E}[\pi_p]}{\partial n} < \frac{\partial \mathbb{E}[\pi_a]}{\partial n} < 0$). This is the case because the effects of more firms on the expected aggregate output and, therefore, the expected price as well as on quantities for both cost situations are more pronounced in the case of perfect observability.

3 Welfare Analysis

In this section, we provide a welfare analysis. To that end, we first decompose expected welfare to highlight the mechanisms which affect it and indicate how these channels depend on the number of firms. Second, we compare expected welfare in case of perfect observability with expected welfare resulting under asymmetric information and show how the number of firms operating in the market affects this comparison.

[8] That is, the set-up features the *competing contract effect* as established by Martimort (1996) for a duopoly. We find that it also arises in oligopoly.

3.1 Decomposition

We start with a general formulation, which encompasses both informational structures. Subsequently, we present the functional forms for each informational setting. Appendix A.4 contains details of the derivations and proofs of claims made in Section 3.1.

3.1.1 A General Formulation

We decompose expected welfare (5) and obtain:

$$\begin{aligned}\mathbb{E}[W_k] = & \underbrace{\int_0^{\mathbb{E}[Q_k]} P(t)dt - \frac{b}{2}n\mathrm{Var}[q_k(\theta)]}_{\text{expected surplus}} \\ & - n\underbrace{\left(\mathbb{E}[\theta]\mathbb{E}[q_k(\theta)] - 2b\mathrm{Var}[q_k(\theta)] + \mathbb{1}_a 2bv\mathrm{Var}[q_a(\theta)]\right)}_{\text{expected production costs } \mathbb{E}[\theta q_k(\theta)]} - \underbrace{nF}_{\text{total sunk costs}}.\end{aligned} \tag{20}$$

We denote with $k \in \{a, p\}$ the respective informational structure and use the indicator function $\mathbb{1}_a$, which equals one in case of asymmetric information and is zero otherwise.

The decomposition clarifies that the variance of aggregate output, $n\mathrm{Var}[q_k(\theta)]$, affects expected welfare via expected surplus and expected production costs. First, expected surplus declines in the variance of aggregate output, as the firms' expected revenues fall by more than expected consumer surplus rises. The negative revenue impact dominates because an expansion in output by low-cost firms, c.p., reduces the price, whereas a decline in output of high-cost firms, c.p., raises the price. Hence, quantity variations are mitigated by price changes in opposite directions, reducing expected revenues by more relative to the rise of expected consumer surplus (see Shapiro, 1986). Therefore, an increase in the variance of aggregate output reduces $\mathbb{E}[W_k]$ via the decline in expected surplus.

Second, an increase in $n\mathrm{Var}[q_k(\theta)]$ reduces expected production costs, $\mathbb{E}[\theta q_k(\theta)]$. Intuitively, this is due to the complementarity between output and ability. An increase in the spread between the quantities, i.e., a rise in the quantity provided by low-cost managers and a fall in the quantity of high-cost managers, results in a more efficient use of resources and, thereby, reduces $\mathbb{E}[\theta q_k(\theta)]$. Consequently, an increase in the variance of aggregate output raises $\mathbb{E}[W_k]$ because of the fall in expected production costs.

The net effect is unambiguously positive for a setting with perfect observability. In the presence of asymmetric information, a higher variance of aggregate output will surely raise expected welfare if the share of low-cost managers, v, is not too high.

3.1.2 Perfect Observability

Using Lemma 1, we can express expected welfare (20) for the case of perfect observability as a function of the number of firms, n:

$$\mathbb{E}[W_p(n)] = \underbrace{\frac{n(a-\mathbb{E}[\theta])(a(2+n)+n\mathbb{E}[\theta])}{2b(1+n)^2} - \underbrace{\frac{nv(1-v)\Delta\theta^2}{8b}}_{\frac{b}{2}\mathrm{Var}[Q_p(n)]}}_{\text{expected surplus}}$$
$$- \underbrace{\left(n\mathbb{E}[\theta]\frac{(a-\mathbb{E}[\theta])}{b(1+n)} - \underbrace{\frac{nv(1-v)\Delta\theta^2}{2b}}_{2b\mathrm{Var}[Q_p(n)]} \right)}_{\text{expected production costs } \mathbb{E}[\theta q_p(\theta,n)]} - nF. \tag{21}$$

The first and the third term in (21) describe the difference between consumer surplus and production costs if marginal costs were non-stochastic and equal to their mean. This difference increases in the number of firms, n, at a decreasing rate because aggregate output is strictly concave in n. This concavity results since a rise in n reduces output per firm. Moreover, aggregate sunk costs, nF, increase linearly in n. Therefore, welfare in a setting in which marginal costs were non-stochastic is strictly concave in the number of firms.[9]

The second and the fourth term in (21) capture the impact of stochastic marginal costs. As clarified in the discussion of (20), their net impact on expected welfare is positive because the reduction in expected production costs dominates the decline in expected surplus. The positive impact of the variance in aggregate output on expected welfare increases linearly with the number of firms.[10] Accordingly, $\mathbb{E}[W_p(n)]$ is also strictly concave in n.

[9] A comparable trade-off determines optimal entry in the settings considered by, for example, Mankiw and Whinston (1986) and Suzumura and Kiyono (1987).

[10] The increase of the variance of aggregate output as the number of firms increases is a consequence of the underlying binomial distribution. In expectation, every additional firm faces a low or a high cost type. With n firms, this scenario is repeated n times, implying that the variance grows boundlessly.

In the subsequent analysis, we assume

$$F > \frac{3v(1-v)\Delta\theta^2}{8b}, \tag{P_3}$$

which ensures that $\mathbb{E}[W_p(n)]$ is not only strictly concave in n, but also that the number of firms, n_p^*, which maximizes expected welfare, solves the respective first-order condition.

3.1.3 Asymmetric Information

In analogy to the case of perfect observability, we can use Lemma 2 and (20) to express $\mathbb{E}[W_a]$ as a function of the number of firms, n:

$$\begin{aligned}\mathbb{E}[W_a(n)] = &\underbrace{\frac{n(a-\overline{\theta})(a(2+n)+n\overline{\theta})}{2b(1+n)^2} - \underbrace{\frac{nv\Delta\theta^2}{8b(1-v)}}_{\frac{b}{2}\mathrm{Var}[Q_a(n)]}}_{\text{expected surplus}} \\ &- \underbrace{\left(n\mathbb{E}[\theta]\frac{(a-\overline{\theta})}{b(1+n)} - \underbrace{n\frac{v\Delta\theta^2}{2b}}_{2b(1-v)\mathrm{Var}[Q_a(n)]} \right)}_{\text{expected production cost } \mathbb{E}[\theta q_a(\theta,n)]} - nF.\end{aligned} \tag{22}$$

$\mathbb{E}[W_a(n)]$ is strictly concave in the number of firms. This curvature reflects the same trade-off as in a setting with perfect observability. Moreover, a higher variance of aggregate output raises expected welfare if $v < 3/4$, capturing the additional impact of stochastic marginal costs in the case of asymmetric information as described by the term $\mathbb{1}_a 2bv\mathrm{Var}[q_a(\theta)]$ in (20).

Finally, we assume

$$F > \frac{v(3-4v)\Delta\theta^2}{8b(1-v)}, \tag{A_3}$$

which ensures that the number of firms, n_a^*, which maximizes $\mathbb{E}[W_a(n)]$ solves the respective first-order condition. Note that if $v < 2/3$, condition (P_3) will always hold, once condition (A_3) is fulfilled.

3.2 Comparison

To compare the levels of expected welfare, which result in the two informational settings, we compute a unique critical number of firms, denoted by $\bar{n}$, for which $\mathbb{E}[W_a(\bar{n})] = \mathbb{E}[W_p(\bar{n})]$ holds. For $v < 2/3$, a positive $\bar{n}$ exists and it is given by [see (21) and (22)]:

$$\overline{n} = \frac{v^2\Delta\theta + 2\sqrt{v(1-v)\Delta\theta(2(a-\overline{\theta})(2-3v) + v\Delta\theta(3-4v))}}{v\Delta\theta(2-3v)}. \tag{23}$$

In Appendix A.5, we show that $\overline{n} > 2$ holds. In addition, we assume that $\overline{n}$ is in the feasible range of firms for both informational settings, i.e., $\overline{n} < n_p^e$ and $\overline{n} < n_a^e$. This will be ensured by a sufficiently high choke price a because the exponent on a in (11), respectively (19), is of higher magnitude than in (23). Moreover, we are able to prove that $n_p^e < n_a^e$ holds.[11] As indicated at the end of Section 2, a given increase in the number of firms reduces quantities more strongly in a world of perfect observability. Hence, the business-stealing externality is more pronounced. In consequence, also the expected price declines by more than in world of asymmetric information. Therefore, expected profits fall more strongly with the number of competitors, n, in a world of perfect observability, and in our set-up $n_p^e < n_a^e$ results. This relationship, in turn, implies that we can restrict the feasible set of firms to $(1, n_p^e]$ when comparing levels of expected welfare.

We also aim to compare expected welfare levels for both informational settings at n_p^*, which is given by:

$$n_p^* = 2\sqrt[3]{\frac{(a-\mathbb{E}[\theta])^2}{8bF - 3v(1-v)\Delta\theta^2}} - 1. \tag{24}$$

The above considerations enable us to state our main results:

Proposition 1.

Assume that $v < 2/3$ holds, i.e., $\bar{n}$ exists.

(i) For $n > \bar{n}$, expected welfare in case of asymmetric information exceeds expected welfare in case of perfect observability, i.e., $\mathbb{E}[W_a(n)] > \mathbb{E}[W_p(n)]$ $\forall$ $n \in (\bar{n}, n_p^e]$.

(ii) For $\overline{n} < n_p^*$, expected welfare in case of asymmetric information exceeds expected

[11] We relegate the respective proof to Appendix A.6 where we present a formal analysis of the model with endogenously determined number of firms.

welfare in case of perfect observability at n_p^*, i.e., $\mathbb{E}[W_a(n_p^*)] > \mathbb{E}[W_p(n_p^*)]$.

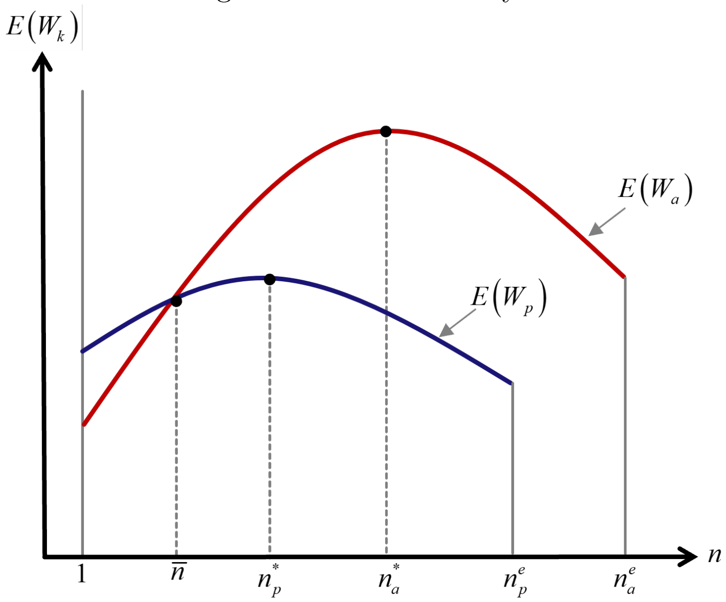
(iii) The number of firms which maximizes expected welfare in case of asymmetric information exceeds the respective number for perfect observability, i.e., $n_a^* > n_p^*$.

Proof Proposition 1.

See Appendix A.5.

Figure 1 illustrates Proposition 1. In a monopoly ($n = 1$), expected welfare is unambiguously lower in the asymmetric information setting than in the case of perfect observability. This is because an owner saves on informational rent payments by lowering the quantity produced by the high-cost manager, while the quantity produced by the low-cost manager remains the same, i.e., there is no distortion at the top.[12]

Figure 1: Welfare Analysis



Moving from a monopoly to an oligopoly, i.e., (marginally) increasing the number of firms, alters the structure of contracts. As also indicated by Etro and Cella (2013), owners set contracts such that the output of the high-cost manager falls by more in a setting with perfect observability than if there is asymmetric information [see (7) and (15)]. Moreover, output of the low-cost manager in a world with asymmetric information rises with the number of firms, relative to the output if there is perfect observability [see (6) and (14)].

[12]For this standard result see, for example, Laffont and Martimort (2002, in particular chapter 2.6.) and the references therein.

Consequently, there are distortions at the bottom and at the top, and the variance of aggregate output rises.

The contractual adjustments have conflicting consequences on expected welfare. First, expected surplus is lower in an oligopoly with asymmetric information than in a world with perfect observability because expected aggregate output is lower, while its variance is higher. This, c.p., reduces $\mathbb{E}[W_a(n)]$ relative to $\mathbb{E}[W_p(n)]$. Second, the increase in the variance of aggregate output reduces expected production costs, as pointed out above. This, c.p., raises the ratio $\mathbb{E}[W_a(n)]/\mathbb{E}[W_p(n)]$. If the probability v of being matched with a low-cost type is not too high ($v < 2/3$), it is possible that the welfare-enhancing effect of a more efficient production outweighs the welfare-reducing impact of a lower expected surplus. This outcome will occur for an oligopoly in which the number of firms exceeds the critical level $\bar{n}$. This threshold exists because $n\text{Var}[q_k(\theta)]$ increases by more with the number of firms in the presence of asymmetric information, implying that expected production costs rise by less with n, relative to the case of perfect observability.

The aforementioned trade-off between a lower expected surplus and more efficient production can imply that expected welfare under asymmetric information exceeds expected welfare under perfect observability even if the latter is maximal. That is, the intersection at $\overline{n}$ is to the left of n_p^*, as illustrated in Figure 1. A comparison of (23) and (24) shows that $\overline{n} < n_p^*$ holds either if $8bF - 3v(1-v)\Delta\theta^2$ is sufficiently small or if the choke price, a, is sufficiently large. The latter is true since the exponent on a in (24) is of higher magnitude than in (23). Intuitively, this scenario is more likely if the negative variance effect on expected surplus is relatively small (because of a relatively high choke price, a), or if its magnitude is large compared to sunk costs, F. In any case, a benevolent social planner would select a greater number of competitors under asymmetric information than in a world of perfect observability, such that n_a^* exceeds n_p^*. This is because the marginal negative impact of a higher variance of aggregate output on expected surplus at n_p^* is still smaller than its marginal effect on expected production costs.

To illustrate Proposition 1 further, we present a numerical example for which all conditions simultaneously hold. The following parameterization is used: $a = 8$, $b = 1$, $v =$

0.15, $\underline{\theta} = 1$, $\overline{\theta} = 2.5$ and $F = 0.124$. Table 1 summarizes our results.[13] Evidently, $\bar{n}$ exists. This implies that expected welfare under asymmetric information exceeds expected welfare in case of perfect observability for all $n \in (10.62, 24.04]$. Moreover, $\bar{n} = 10.62 < 11.59 = n_p^*$ holds as well, which implies $\mathbb{E}[W_a(n_p^*)] > \mathbb{E}[W_p(n_p^*)]$ in accordance with Proposition 1.

Table 1: Numerical Example A – Basic Model

Example	$\bar{n}$	n_p^*	n_a^*	n_p^e	n_a^e
A	10.62	11.59	21.89	24.04	33.97

4 Extension: Firm Entry

In this section, we assume that the number of firms is determined endogenously to compare the incentives to enter the market in equilibrium for both informational settings with the respective outcomes a social planner would prefer. In addition, we analyze whether part *(i)* of Proposition 1 holds for the number of firms resulting in market equilibrium.[14] To extend our framework accordingly, we assume that the entry decision takes place before managers realize their type.

4.1 Excessive and Insufficient Entry

In a market equilibrium, a firm will enter the market as long as expected profits are non-negative, such that $\mathbb{E}[\pi_k(n_k^e)] = 0$ holds. The social planner chooses the number of firms, n_k^*, to maximize expected welfare, $\mathbb{E}[W_k(n)]$, while firms select output levels.

In a world of certainty, the number of entrants into a homogeneous Cournot-oligopoly will be excessive if there is business stealing (excessive entry prediction) (see Perry, 1984, Mankiw and Whinston, 1986, Suzumura and Kiyono, 1987, von Weizsäcker, 1980, Amir et al., 2014). In our model, marginal costs are stochastic, in contrast to the original contributions. Since information is asymmetrically distributed in one of the settings, our

[13] For simplicity, in our numerical examples, we round up or down numbers.

[14] We are extremely grateful to an anonymous referee for suggesting this extension of our base model. Parts *(ii)* and *(iii)* of Proposition 1 focus on the welfare-maximizing number of firms and are, therefore, unaffected by the possibility of firm entry.

approach features a further distortion. Hence, we investigate whether the excessive entry prediction occurs in the present framework as well.

To do so, we focus on the relationship between expected welfare and the number of firms for each informational setting separately. Differentiating the social planner's objective function and evaluating the result at the number of firms, n_k^e, which enter in market equilibrium, we obtain (see Appendix A.6 for details of the derivation):

$$\left.\frac{\mathrm{d}\mathbb{E}[W_k]}{\mathrm{d}n}\right|_{n=n_k^e} = \left.\left(n(\mathbb{E}[P_k] - \mathbb{E}[\theta])\frac{\mathrm{d}\mathbb{E}[q_k(\theta)]}{\mathrm{d}n} + \frac{b}{2}\mathrm{Var}[q_k(\theta)] + \mathbb{1}_a\mathbb{E}[rent_a]\right)\right|_{n=n_k^e}. \tag{25}$$

The first term in (25) measures the strength of the business stealing externality. In addition to this standard effect, the social planner's evaluation of the number of firms in market equilibrium is determined by the variance of output and, in case of asymmetric information, additionally by the expected rent.

The business stealing externality gives rise to excessive entry because firms do not take into account that they reduce output of competitors, for which consumers are willing to pay a price in excess of expected marginal production costs. The variance of aggregate output, $n\mathrm{Var}[q_k(\theta)]$, raises expected consumer surplus. Firms do not incorporate this effect when deciding about market entry and therefore, c.p., face insufficient incentives. The expected rent in a world of asymmetric information deters entry on account of its profit-reducing impact but does not affect the socially optimal number of firms. If the impact of the variance and, in case of asymmetric information, of the expected rent dominates the business stealing externality, there is insufficient entry. Therefore, the excessive entry prediction does not necessarily hold in our framework (see Appendix A.6 for the proof).[15]

In the numerical example A (see Table 1), entry is excessive in both informational settings, i.e., the business stealing effect outweighs the variance and the rent effect. To illustrate the possibility of insufficient entry, we modify the example (see Table 2). In particular, we decrease the choke price to $a = 3$ (instead of assuming $a = 8$), because this lowers the magnitude of the business stealing effect, and retain all other parameter values.

[15]This is also reflected in the findings from other contributions, which consider different forms of uncertainty and can either establish the possibility of insufficient entry (Creane, 2007, Deo and Corbett, 2009, Jansen and Özaltın, 2018) or show that the business stealing externality dominates (de Pinto and Goerke, 2022).

As a result, entry is insufficient in both informational settings ($n_p^e = 2.171 < 2.175 = n_p^*$ and $n_a^e = 2.179 < 4.66 = n_a^*$).

Table 2: Numerical Example B – Insufficient Entry

Example	n_p^*	n_a^*	n_p^e	n_a^e
B	2.175	4.66	2.171	2.179

4.2 Robustness of Proposition 1 Part *(i)*

Proposition 1 part *(i)* asserts that for all $n \in (\bar{n}, n_p^e]$, expected welfare in case of asymmetric information exceeds expected welfare in case of perfect observability. Therefore, it follows immediately that $\mathbb{E}[W_a(n_p^e)] > \mathbb{E}[W_p(n_p^e)]$ holds. Regarding n_a^e, we cannot compare $\mathbb{E}[W_a(n_a^e)]$ to $\mathbb{E}[W_p(n_a^e)]$, as the restriction of non-negative expected profits would be violated in the latter case because $n_p^e < n_a^e$ holds. However, we can inquire whether the welfare ranking also applies if we compare expected welfare levels in both informational settings at the respective endogenously determined number of firms. Put differently, we investigate whether $\mathbb{E}[W_a(n_a^e)] > \mathbb{E}[W_p(n_p^e)]$ is feasible.

The answer will not necessarily be in the affirmative because $\mathbb{E}[W_a(n_a^e)]$ may fall below $\mathbb{E}[W_p(n_p^e)]$ due to the feature that expected welfare is inverted U-shaped in n, as elucidated in Section 3. In Appendix A.6, we derive a condition, which ensures that $\mathbb{E}[W_a(n_a^e)] > \mathbb{E}[W_p(n_p^e)]$ can hold. Compared to our basic model with an exogenously given number of firms, the requirement is more restrictive. In particular, the share of low-cost managers has to be lower than the value postulated in Proposition 1, while the choke price, a, must still be sufficiently high. Using the numerical example A (see Table 1), based on $v = 0.15$, we obtain $\mathbb{E}[W_a(n_a^e)] = 16.15 > 15.97 = \mathbb{E}[W_p(n_p^e)]$. This example indicates the robustness of Proposition 1 part *(i)*.

Intuitively, the same trade-off, namely lower aggregate output and higher variance of aggregate output in case of asymmetric information, is also present in this scenario. However, since expected profits are zero, expected consumer surplus is the main driver of the result, implying that the welfare-enhancing effect of the variance must be more pronounced than in a setting with strictly positive expected profits. This can be achieved by a sufficiently small share of low-costs managers.

5 Conclusion

Our paper assumes that a principal-agent relationship in the production sphere co-exists with a Cournot-oligopoly on the output market. We distinguish two informational settings. In a situation with perfect observability, an owner observes the type of manager matched with at the contracting stage. In case of asymmetric information, only the manager is aware of the own type.

As our main result, we establish the existence of a separating equilibrium, which can result in a higher level of expected welfare in case of asymmetric information. Expected welfare in a setting with asymmetric information may even exceed the maximum of expected welfare obtainable in a world with perfect observability. Moreover, such a welfare ranking can also arise if the number of firms is determined endogenously by a profit-constraint. In short: Having less knowledge can raise expected welfare.

From the perspective of society, given that oligopoly is a typical market structure, our results suggest that policy measures which aim at enhancing welfare should account for informational frictions, as they are a widespread phenomenon. In addition, our paper provides another example for a second-best world.

A Appendix

A.1 Findings for the Setting with Perfect Observability

Proof of Lemma 1.

Under perfect observability of a manager's marginal costs, each firm owner solves the following problem:

$$
\begin{aligned}
&\max_{\{\underline{q},\underline{w},\overline{q},\overline{w}\}} \mathbb{E}[\pi_p] \\
&s.t. \\
&U\left(\underline{w},\underline{q},\underline{\theta}\right) = \underline{w} - \underline{\theta}\underline{q} \geqslant 0, && PC(\underline{\theta}) \\
&U\left(\overline{w},\overline{q},\overline{\theta}\right) = \overline{w} - \overline{\theta}\overline{q} \geqslant 0. && PC(\overline{\theta})
\end{aligned}
$$

Since types are observable, both participation constraints bind. Using wages for the binding PCs in the objective function, the maximization problem consists of finding quantities:

$$
\underset{\{\underline{q},\overline{q}\}}{Max}\, \mathbb{E}[\pi_p] = \begin{pmatrix} v\left[(a - b\underline{q} - b\mathbb{E}[Q_{-i}] - \underline{\theta})\underline{q}\right] + \\ (1-v)\left[(a - b\overline{q} - b\mathbb{E}[Q_{-i}] - \overline{\theta})\overline{q}\right] \end{pmatrix} - F. \tag{A.1}
$$

Computing the first-order conditions, which are also sufficient, yields:

$$
\frac{\partial \mathbb{E}[\pi_p]}{\partial \underline{q}} = 0 \Rightarrow -b\underline{q} + (a - b\underline{q} - b\mathbb{E}[Q_{-i}] - \underline{\theta}) = 0, \tag{A.2}
$$

$$
\frac{\partial \mathbb{E}[\pi_p]}{\partial \overline{q}} = 0 \Rightarrow -b\overline{q} + (a - b\overline{q} - b\mathbb{E}[Q_{-i}] - \overline{\theta}) = 0. \tag{A.3}
$$

Together with $\mathbb{E}[Q_{-i}]$, we have a linear system with three unknowns:

$$
\begin{cases}
-b\underline{q} + (a - b\underline{q} - b\mathbb{E}[Q_{-i}] - \underline{\theta}) = 0 \\
-b\overline{q} + (a - b\overline{q} - b\mathbb{E}[Q_{-i}] - \overline{\theta}) = 0 \\
\mathbb{E}[Q_{-i}] = (n-1)[v\underline{q} + (1-v)\overline{q}].
\end{cases} \tag{A.4}
$$

It is straightforward to obtain the solutions:

$$
\underline{q}_p = \frac{a - \underline{\theta}}{b(1+n)} + \frac{(n-1)(1-v)}{2b(1+n)}\Delta\theta, \tag{A.5}
$$

$$
\overline{q}_p = \frac{a - \overline{\theta}}{b(1+n)} - \frac{(n-1)v}{2b(1+n)}\Delta\theta. \tag{A.6}
$$

Then, expected aggregate output is:

$$\mathbb{E}[Q_p] = n[v\underline{q}_p + (1-v)\overline{q}_p] = \frac{n(a - \mathbb{E}[\theta])}{b(1+n)}. \tag{A.7}$$

Given the demand schedule, we find:

$$\mathbb{E}[P_p] = a - b\mathbb{E}[Q_p] = \frac{a + n\mathbb{E}[\theta]}{1+n}. \tag{A.8}$$

Using (A.5) and (A.6), we compute:

$$Var[q_p(\theta)] = \mathbb{E}[(q_p(\theta))^2] - \mathbb{E}[q_p(\theta)]^2 = \frac{v(1-v)\Delta\theta^2}{4b^2}. \tag{A.9}$$

Using (A.2) and (A.3), we can express expected profits [see (A.1)] as:

$$\begin{aligned}\mathbb{E}[\pi_p] &= b[v\underline{q}_p^2 + (1-v)\overline{q}_p^2] - F = b\mathbb{E}[(q_p(\theta))^2] - F \\ &= \frac{(a - \mathbb{E}[\theta])^2}{b(1+n)^2} + \frac{v(1-v)\Delta\theta^2}{4b} - F.\end{aligned} \tag{A.10}$$

This proves Lemma 1. ■

The effect of n on quantities

From equations (A.5) and (A.6), we obtain:

$$\frac{\partial \underline{q}_p}{\partial n} = \frac{-(a - \underline{\theta})}{b(1+n)^2} + \frac{(1-v)\Delta\theta}{b(1+n)^2} = \frac{-(a-\overline{\theta}) - v\Delta\theta}{b(1+n)^2} = \frac{-(a-\overline{\theta})}{b(1+n)^2} - \frac{v\Delta\theta}{b(1+n)^2} = \frac{\partial \overline{q}_p}{\partial n} < 0. \tag{A.11}$$

The inequalities follow from the assumption $\overline{q}_p(n) > 0$, which necessarily requires $a - \overline{\theta} > 0$ (see (A.6)).

Condition (P_2)

To obtain (P_2), notice from (A.6) that

$$\begin{aligned}\overline{q}_p =& \frac{a-\overline{\theta}+v\Delta\theta-v\Delta\theta}{b(1+n)} - \frac{(n-1)v}{2b(1+n)}\Delta\theta = \\ & \frac{a-\mathbb{E}[\theta]}{b(1+n)} - \frac{v\Delta\theta}{b(1+n)} - \frac{(n-1)v}{2b(1+n)}\Delta\theta = \frac{a-\mathbb{E}[\theta]}{b(1+n)} - \frac{v\Delta\theta}{2b}.\end{aligned} \tag{A.12}$$

Furthermore, equation (11) can be rewritten as:

$$\frac{a-\mathbb{E}[\theta]}{1+n_p^e} = \sqrt{b\left(F - \frac{v(1-v)\Delta\theta^2}{4b}\right)}. \tag{A.13}$$

Evaluating $\overline{q}_p$ as expressed in equation (A.12) at n_p^e, we obtain

$$\begin{aligned}&\overline{q}_p(n_p^e) = \frac{a-\mathbb{E}[\theta]}{b(1+n_p^e)} - \frac{v\Delta\theta}{2b} > 0 \iff \\ &\frac{1}{b}\sqrt{b\left(F - \frac{v(1-v)\Delta\theta^2}{4b}\right)} - \frac{v\Delta\theta}{2b} > 0.\end{aligned} \tag{A.14}$$

Solving this last inequality for F, we obtain (P_2).

A.2 Findings for the Setting with Asymmetric Information

Proof of Lemma 2.

To derive the menu of contracts that each owner offers, we follow Laffont and Martimort (2002) and express everything in terms of informational rents:

$$\underline{U} = \underline{w} - \underline{\theta}\underline{q}, \tag{A.15}$$

$$\overline{U} = \overline{w} - \overline{\theta}\overline{q}. \tag{A.16}$$

The (PC) constraints are $\underline{U} \geqslant 0$, $\overline{U} \geqslant 0$, while the (IC) constraints read:

$$\begin{aligned}&\underline{w} - \underline{\theta}\underline{q} \geqslant \overline{w} - \underline{\theta}\overline{q} \\ \Rightarrow &\underline{U} \geqslant \overline{U} + \Delta\theta\overline{q},\end{aligned} \tag*{$IC\,(\underline{\theta})$}$$

$$
\begin{aligned}
&\overline{w} - \overline{\theta}\overline{q} \geqslant \underline{w} - \overline{\theta}\underline{q} \\
\Rightarrow &\overline{U} \geqslant \underline{U} - \Delta\theta\underline{q}.
\end{aligned}
\qquad IC\left(\overline{\theta}\right)
$$

After substitutions of wages from (A.15) and (A.16), the maximization problem reads:

$$
\begin{aligned}
&\underset{\{\underline{q},\underline{U},\overline{q},\overline{U}\}}{Max}\ \mathbb{E}[\pi_a] = \begin{pmatrix} v\left[(a - b\underline{q} - b\mathbb{E}[Q_{-i}] - \underline{\theta})\underline{q}\right] + \\ (1-v)\left[(a - b\overline{q} - b\mathbb{E}[Q_{-i}] - \overline{\theta})\overline{q}\right] \end{pmatrix} - (v\underline{U} + (1-v)\overline{U}) - F \qquad \text{(A.17)} \\
&s.t. \quad PC\left(\underline{\theta}\right), PC\left(\overline{\theta}\right), IC\left(\underline{\theta}\right), IC\left(\overline{\theta}\right).
\end{aligned}
$$

As it is standard, $IC(\underline{\theta})$ and $PC(\overline{\theta})$ imply $PC(\underline{\theta})$. Disregarding $IC(\overline{\theta})$ and checking it at the optimum, the remaining two constraints are $PC(\overline{\theta})$ and $IC(\underline{\theta})$, which have to bind at the optimum. As a consequence, the relaxed problem is:

$$
\underset{\{\underline{q},\overline{q}\}}{Max}\,\mathbb{E}\left[\pi_a\right] = v(a - b\underline{q} - b\mathbb{E}[Q_{-i}] - \underline{\theta})\underline{q} + (1-v)\left(a - b\overline{q} - b\mathbb{E}[Q_{-i}] - \overline{\theta}\right)\overline{q} - v\Delta\theta\overline{q} - F. \qquad \text{(A.18)}
$$

Computing the first-order conditions, which are also sufficient, yields:

$$
\begin{aligned}
\frac{\partial\mathbb{E}[\pi_a]}{\partial\underline{q}} &= 0 \Rightarrow -b\underline{q} + (a - b\underline{q} - b\mathbb{E}[Q_{-i}] - \underline{\theta}) = 0, && \text{(A.19)} \\
\frac{\partial\mathbb{E}[\pi_a]}{\partial\overline{q}} &= (1-v)[-b\overline{q} + (a - b\overline{q} - b\mathbb{E}[Q_{-i}] - \overline{\theta})] - v\Delta\theta = 0. && \text{(A.20)}
\end{aligned}
$$

Together with $\mathbb{E}[Q_{-i}]$, we have a linear system with three unknowns:

$$
\begin{cases}
-b\underline{q} + (a - b\underline{q} - b\mathbb{E}[Q_{-i}] - \underline{\theta}) = 0 \\
(1-v)[-b\overline{q} + (a - b\overline{q} - b\mathbb{E}[Q_{-i}] - \overline{\theta})] - v\Delta\theta = 0 \\
\mathbb{E}[Q_{-i}] = (n-1)[v\underline{q} + (1-v)\overline{q}].
\end{cases}
\qquad \text{(A.21)}
$$

Hence, we obtain:

$$\underline{q}_a = \frac{a-\underline{\theta}}{b(1+n)} + \frac{n-1}{2b(1+n)}\Delta\theta, \tag{A.22}$$

$$\overline{q}_a = \frac{a-\overline{\theta}}{b(1+n)} - \frac{1}{2b}\frac{v}{1-v}\Delta\theta, \tag{A.23}$$

$$\mathbb{E}[Q_a] = \frac{n(a-\overline{\theta})}{b(1+n)}, \tag{A.24}$$

$$\mathbb{E}[P_a] = \frac{a+n\overline{\theta}}{1+n}. \tag{A.25}$$

Note that $IC(\overline{\theta})$ is slack because $\underline{q}_a(n) > \overline{q}_a(n)$ for every $n \geqslant 1$. Using (A.22) and (A.23), we compute:

$$Var[q_a(\theta)] = \mathbb{E}[(q_a(\theta))^2] - \mathbb{E}[q_a(\theta)]^2 = \frac{v\Delta\theta^2}{(1-v)4b^2}. \tag{A.26}$$

Using (A.19) and (A.20), we can express expected profits [see(A.18)] as:

$$\begin{aligned}\mathbb{E}[\pi_a] &= b[v\underline{q}_a^2 + (1-v)\overline{q}_a^2] - F = b\mathbb{E}[(q_a(\theta))^2] - F\\ &= \frac{(a-\overline{\theta})^2}{b(1+n)^2} + \frac{v\Delta\theta^2}{4b(1-v)} - F.\end{aligned} \tag{A.27}$$

This proves Lemma 2. ■

The effect of n on quantities

From equations (A.22) and (A.23), we obtain:

$$\frac{\partial \underline{q}_a}{\partial n} = -\frac{a-\underline{\theta}}{b(1+n)^2} + \frac{\Delta\theta}{b(1+n)^2} = \frac{-(a-\overline{\theta})}{b(1+n)^2} = \frac{\partial \overline{q}_a}{\partial n} < 0. \tag{A.28}$$

The inequalities follow from the assumption $\overline{q}_a(n) > 0$, which necessarily requires $a - \overline{\theta} > 0$ (see (A.23)).

Condition (A_2)

To obtain (A_2), notice that (19) can be rewritten as:

$$\frac{a-\overline{\theta}}{1+n_a^e} = \sqrt{b\left(F - \frac{v\Delta\theta^2}{4b(1-v)}\right)}. \tag{A.29}$$

Evaluating $\overline{q}_a$ as expressed in equation (A.23) at n_a^e, we obtain

$$\begin{aligned} &\overline{q}_a(n_a^e) = \frac{a-\overline{\theta}}{b(1+n_a^e)} - \frac{1}{2b}\frac{v}{1-v}\Delta\theta > 0 \iff \\ &\frac{1}{b}\sqrt{b\left(F - \frac{v\Delta\theta^2}{4b(1-v)}\right)} - \frac{1}{2b}\frac{v}{1-v}\Delta\theta > 0. \end{aligned} \tag{A.30}$$

Solving this last inequality for F, we obtain (A_2).

A.3 Comparing the Impact of n for both Informational Settings

From equations (A.11) and (A.28), it is straightforward to establish:

$$\frac{\partial \underline{q}_p}{\partial n} = \frac{\partial \overline{q}_p}{\partial n} < \frac{\partial \underline{q}_a}{\partial n} = \frac{\partial \overline{q}_a}{\partial n} < 0. \tag{A.31}$$

Moreover, from (A.7), (A.24), and the linear inverse demand function, we obtain

$$\frac{\partial \mathbb{E}[Q_p]}{\partial n} = \frac{a-\mathbb{E}[\theta]}{b(1+n)^2} > \frac{\partial \mathbb{E}[Q_a]}{\partial n} = \frac{a-\overline{\theta}}{b(1+n)^2} > 0 \tag{A.32}$$

and

$$\frac{\partial \mathbb{E}[P_p]}{\partial n} = -b\frac{\partial \mathbb{E}[Q_p]}{\partial n} < \frac{\partial \mathbb{E}[P_a]}{\partial n} = -b\frac{\partial \mathbb{E}[Q_a]}{\partial n}. \tag{A.33}$$

A.4 Formal Aspects of the Welfare Analysis

Derivation of Equation (20)

Using the variance equality, expected welfare (5) can be written as:

$$\begin{aligned} \mathbb{E}[W_k] &= \mathbb{E}[(a - \frac{b}{2}Q_k)Q_k - n\theta q_k(\theta) - nF] \\ &= a\mathbb{E}[Q_k] - \frac{b}{2}\mathbb{E}[Q_k]^2 - \frac{b}{2}n\mathrm{Var}[q_k(\theta)] - n\mathbb{E}[\theta q_k(\theta)] - nF \\ &= \int_0^{\mathbb{E}[Q_k]} P(t)dt - \frac{b}{2}n\mathrm{Var}[q_k(\theta)] - n\mathbb{E}[\theta q_k(\theta)] - nF. \end{aligned} \tag{A.34}$$

Expected productions costs, expected unit production costs and expected output read:

$$\mathbb{E}[\theta q_k(\theta)] = v\underline{\theta}\underline{q}_k + (1-v)\overline{\theta}\overline{q}_k, \tag{A.35}$$

$$\mathbb{E}[\theta] = v\underline{\theta} + (1-v)\overline{\theta}, \tag{A.36}$$

$$\mathbb{E}[q_k(\theta)] = v\underline{q}_k + (1-v)\overline{q}_k, \tag{A.37}$$

respectively. Expanding (A.35), as well as using (A.36) and (A.37), we obtain:

$$\begin{aligned}\mathbb{E}[\theta q_k(\theta)] &= v\underline{\theta}\underline{q}_k + (1-v)\overline{\theta}\overline{q}_k + \mathbb{E}[\theta]\mathbb{E}[q_k(\theta)] - \mathbb{E}[\theta]\mathbb{E}[q_k(\theta)] \\ &= \mathbb{E}[\theta]\mathbb{E}[q_k(\theta)] - v(1-v)\Delta\theta(\underline{q}_k - \overline{q}_k).\end{aligned} \tag{A.38}$$

From (A.5) and (A.6), as well as (A.22) and (A.23), we derive $\underline{q}_p - \overline{q}_p = \frac{\Delta\theta}{2b}$ and $\underline{q}_a - \overline{q}_a = \frac{\Delta\theta}{2b(1-v)}$, respectively. Combining these output differences with (A.9) and (A.26), we can substitute the last term in (A.38) and obtain:

$$\mathbb{E}[\theta q_k(\theta)] = \mathbb{E}[\theta]\mathbb{E}[q_k(\theta)] - 2b\mathrm{Var}[q_k(\theta)] + \mathbb{1}_a 2bv\mathrm{Var}[q_a(\theta)]. \tag{A.39}$$

Inserting (A.39) into (A.34) yields (20).

Expected Welfare: First and Second-Order Derivatives

Differentiating (21) and (22) yields:

$$\frac{\mathrm{d}\mathbb{E}[W_p]}{\mathrm{d}n} = \frac{(a-\mathbb{E}[\theta])^2}{b(1+n)^3} + \frac{3v(1-v)\Delta\theta^2}{8b} - F, \tag{A.40}$$

$$\frac{\mathrm{d}^2\mathbb{E}[W_p]}{\mathrm{d}n^2} = -3\frac{(a-\mathbb{E}[\theta])^2}{b(1+n)^4} < 0, \tag{A.41}$$

$$\frac{\mathrm{d}\mathbb{E}[W_a]}{\mathrm{d}n} = \frac{(a-\overline{\theta})(a-\overline{\theta}+v\Delta\theta(1+n))}{b(1+n)^3} + \frac{v(3-4v)\Delta\theta^2}{8b(1-v)} - F, \tag{A.42}$$

$$\frac{\mathrm{d}^2\mathbb{E}[W_a]}{\mathrm{d}n^2} = -(a-\overline{\theta})\frac{[3(a-\overline{\theta})+2(1+n)v\Delta\theta]}{b(1+n)^4} < 0. \tag{A.43}$$

Conditions (P_3) and (A_3) are derived from (A.40) and (A.42), respectively. Moreover, (A.41) and (A.43) show the strict concavity of expected welfare, such that the maximizer

$n_k^* \triangleq \text{argmax}_n \, \mathbb{E}[W_k(n_k)]$ is unique.

A.5 Proof of Proposition 1

To prove part *(i)*, we use (21) and (22) and obtain for $n = 1$:

$$\mathbb{E}[W_a(1)] - \mathbb{E}[W_p(1)] = -\frac{v\Delta\theta}{8b(1-v)}(2(1-v)(a-\overline{\theta}) + v\Delta\theta) < 0. \tag{A.44}$$

Because $\mathbb{E}[W_p(n)]$ and $\mathbb{E}[W_a(n)]$ intersect only once at $n = \overline{n}$, we then have $\mathbb{E}[W_a(n)] > \mathbb{E}[W_p(n)] \; \forall \; n \in (\bar{n}, n_p^e]$, as claimed in the first part of the Proposition.

Since $\overline{n}$ and n_k^* are unique, we immediately find that for $n_p^* > \overline{n}$, $\mathbb{E}[W_a(n_p^*)] > \mathbb{E}[W_p(n_p^*)]$ holds. This proves part *(ii)* of the Proposition.

To prove part *(iii)*, we proceed in three steps. First, we show that $\overline{n} > 2 \; \forall \; v \in (0, 2/3)$, and $\overline{n} > 3 \; \forall \; v \in [1/2, 2/3)$. Second, we use the assumption $n_p^e > \overline{n}$ to prove $n_p^* > 1 \; \forall \; v \in (0, 2/3)$, and that $n_p^* > 3/2 \; \forall \; v \in [1/2, 2/3)$. Third, we observe that the derivative $\mathrm{d}\mathbb{E}[W_a(n)]/\mathrm{d}n$ is positive when evaluated at n_p^*. Given strict concavity of $\mathbb{E}[W_k]$, this implies then $n_a^* > n_p^*$.

Step 1.

Rearranging (23) yields:

$$\overline{n} = \frac{v}{(2-3v)} + 2\sqrt{\frac{v(1-v)\Delta\theta(2(a-\overline{\theta})(2-3v) + v\Delta\theta(3-4v))}{v^2\Delta\theta^2(2-3v)^2}}. \tag{A.45}$$

To prove $\overline{n} > 2$, it is sufficient to show that the term under the square root in (A.45) exceeds one. This will be true if:

$$\begin{aligned} & v(1-v)\Delta\theta(2(a-\overline{\theta})(2-3v) + v\Delta\theta(3-4v)) > v^2\Delta\theta^2(2-3v)^2 \\ \Leftrightarrow & (2-3v)[2(1-v)(a-\overline{\theta}) - v\Delta\theta(2-3v)] + v(1-v)\Delta\theta(3-4v) > 0. \end{aligned} \tag{A.46}$$

Since $v < 2/3$, $\overline{n} > 2$ holds if the term in square brackets is positive. That is, if:

$$a - \overline{\theta} - \frac{v}{1-v}\Delta\theta\frac{2-3v}{2} > 0. \tag{A.47}$$

Using (A.23), $\overline{q}_a > 0$ and $n > 1$, we see that the inequality (A.47) is fulfilled. Accordingly, $\overline{n} > 2$ holds. Moreover, we get $\overline{n} > 3$ if $1/2 \leqslant v < 2/3$ because the first term in (A.45) is not smaller than 1.

Step 2.

Because of our assumption that $\overline{n}$ is in the feasible set of firms, we get $2 < \bar{n} < n_p^e$ if $0 < v < 2/3$. We can then use (11) to obtain:

$$a - \mathbb{E}[\theta] > \frac{3}{2}\sqrt{4bF - v(1-v)\Delta\theta^2}. \tag{A.48}$$

Combining (A.48) and (24) leads to:

$$n_p^* = 2\sqrt[3]{\frac{(a-\mathbb{E}[\theta])^2}{8bF - 3v(1-v)\Delta\theta^2}} - 1 > 2\sqrt[3]{\frac{(9/4)(4bF - v(1-v)\Delta\theta^2)}{8bF - 3v(1-v)\Delta\theta^2}} - 1 > 1. \tag{A.49}$$

For $1/2 \leqslant v < 2/3$, analogously, $3 < \overline{n} < n_p^e$ and (11) imply:

$$a - \mathbb{E}[\theta] > 2\sqrt{4bF - v(1-v)\Delta\theta^2}, \tag{A.50}$$

which results in [see (24)]:

$$n_p^* = 2\sqrt[3]{\frac{(a-\mathbb{E}[\theta])^2}{8bF - 3v(1-v)\Delta\theta^2}} - 1 > 2\sqrt[3]{\frac{4(4bF - v(1-v)\Delta\theta^2)}{8bF - 3v(1-v)\Delta\theta^2}} - 1 > \frac{3}{2}. \tag{A.51}$$

Step 3.

To prove that the derivative $\mathrm{d}\mathbb{E}[W_a(n)]/\mathrm{d}n$ is positive when evaluated at n_p^*, consider first the following expression obtained from (A.40) and (A.42):

$$\left[\frac{\mathrm{d}\mathbb{E}[W_a(n)]}{\mathrm{d}n} - \frac{\mathrm{d}\mathbb{E}[W_p(n)]}{\mathrm{d}n}\right]\Bigg|_{n=n_p^*} = \frac{v\Delta\theta(n_p^*-1)(a-\overline{\theta})}{b(1+n_p^*)^3} - \frac{v^2\Delta\theta^2}{b(1+n_p^*)^3} + \frac{v^2\Delta\theta^2(2-3v)}{8b(1-v)}. \tag{A.52}$$

If $1/2 \leqslant v < 2/3$, we have $n_p^* > 3/2$. Using (A.23), and $\overline{q}_a > 0$, we find:

$$\frac{a-\overline{\theta}}{2} > \frac{v}{4(1-v)}\Delta\theta(1+n_p^*) > v\Delta\theta. \tag{A.53}$$

Therefore, (A.52) has a positive sign.

If $0 < v < 2/3$, we have $n_p^* > 1$. Inserting the lower bound of n_p^* into the negative term in (A.52) yields:

$$\left[\frac{\mathrm{d}\mathbb{E}[W_a(n)]}{\mathrm{d}n} - \frac{\mathrm{d}\mathbb{E}[W_p(n)]}{\mathrm{d}n}\right]\Bigg|_{n=n_p^*} > \frac{v\Delta\theta(n_p^*-1)(a-\overline{\theta})}{b(1+n_p^*)^3} - \frac{v^2\Delta\theta^2}{8b} + \frac{v^2\Delta\theta^2(2-3v)}{8b(1-v)}$$
$$= \frac{v\Delta\theta(n_p^*-1)(a-\overline{\theta})}{b(1+n_p^*)^3} + \frac{v^2\Delta\theta^2(1-2v)}{8b(1-v)} > 0, \tag{A.54}$$

which completes the proof of part *(iii)* of the Proposition.

A.6 Formal Aspects of the Extension with Firm Entry

Derivation of Equation (25)

Differentiating (20) yields:

$$\begin{aligned}\frac{\mathrm{d}\mathbb{E}[W_k]}{\mathrm{d}n} =&(a-b\mathbb{E}[Q_k])\mathbb{E}[q_k(\theta)] + n(a-b\mathbb{E}[Q_k])\frac{\mathrm{d}\mathbb{E}[q_k(\theta)]}{\mathrm{d}n} - n\frac{\mathrm{d}\mathbb{E}[\theta q_k(\theta)]}{\mathrm{d}n} \\ &- \mathbb{E}[\theta q_k(\theta)] - \frac{b}{2}\mathrm{Var}[q_k(\theta)] - F.\end{aligned} \tag{A.55}$$

Rewriting (3) leads to:

$$\begin{aligned}\mathbb{E}\left[\pi_k\right] &= v[(a-b\underline{q}_k - b\mathbb{E}[Q_{k,-i}] - \underline{\theta})\underline{q}_k] + (1-v)[(a-b\overline{q}_k - b\mathbb{E}[Q_{k,-i}] - \overline{\theta})\overline{q}_k] \\ &\quad - \mathbb{1}_a\mathbb{E}[rent_a] - F \\ &= (a-b\mathbb{E}[Q_k])\mathbb{E}[q_k(\theta)] - \mathbb{E}[\theta q_k(\theta)] - b\mathrm{Var}[q_k(\theta)] - \mathbb{1}_a\mathbb{E}[rent_a] - F.\end{aligned} \tag{A.56}$$

Note that we used $\overline{w}_k = \overline{\theta}\overline{q}_k$ and $\underline{w}_p = \underline{\theta}\underline{q}_p$ (due to the binding $PC(\theta)$) and $\underline{w}_a = \underline{\theta}\underline{q}_a + \Delta\theta\overline{q}_a$ (due to the binding $IC(\underline{\theta})$) to obtain the first line and the variance equality as well as $\mathbb{E}[Q] = \mathbb{E}[Q_{-i}] + \mathbb{E}[q(\theta)]$ to obtain the second.

Using $\mathbb{E}\left[\pi_k\right] = 0$ and (A.56), we can evaluate (A.55) at n_k^e to find:

$$\frac{\mathrm{d}\mathbb{E}[W_k]}{\mathrm{d}n}\Bigg|_{n=n_k^e} = \left(n(a-b\mathbb{E}[Q_k])\frac{\mathrm{d}\mathbb{E}[q_k(\theta)]}{\mathrm{d}n} - n\frac{\mathrm{d}\mathbb{E}[\theta q_k(\theta)]}{\mathrm{d}n} + \frac{b}{2}\mathrm{Var}[q_k(\theta)] + \mathbb{1}_a\mathbb{E}[rent_a]\right)\Bigg|_{n=n_k^e}. \tag{A.57}$$

Employing $\frac{\mathrm{d}\mathbb{E}[\theta q(\theta)]}{\mathrm{d}n} = \frac{\mathrm{d}\mathbb{E}[q(\theta)]}{\mathrm{d}n}\mathbb{E}[\theta]$ and $a - b\mathbb{E}[Q_k] = \mathbb{E}[P_k]$, we can rearrange (A.57) to obtain (25).

A Remark on the Conditions

For later use, we observe that (P_2) neither implies (P_3) nor vice versa, but that their relation depends on v. Therefore, we can write:

$$F > \max\left\{\underbrace{\frac{3v(1-v)\Delta\theta^2}{8b}}_{v<\frac{1}{3}}, \underbrace{\frac{v\Delta\theta^2}{4b}}_{v>\frac{1}{3}}\right\}. \qquad (P_2 + P_3)$$

For (A_2) and (A_3), we find:

$$F > \max\left\{\underbrace{\frac{v(3-4v)\Delta\theta^2}{8b(1-v)}}_{v<\frac{1}{8}(7-\sqrt{33})}, \underbrace{\frac{v\Delta\theta^2}{4b(1-v)^2}}_{v>\frac{1}{8}(7-\sqrt{33})}\right\}. \qquad (A_2 + A_3)$$

If $v < 2/3$, ($A_2 + A_3$) implies ($P_2 + P_3$).

Excessive and Insufficient Entry: Perfect Observability

To compare the number of firms in market equilibrium with the socially optimal number of firms in a world with perfect observability, we use Lemma 1 and rewrite (25) as:

$$\phi_p \equiv \left.\frac{\mathrm{d}\mathbb{E}[W_p(n)]}{\mathrm{d}n}\right|_{n=n_p^e} = \frac{1}{8b}\left(-8bF + 3v(1-v)\Delta\theta^2 + \frac{(4bF - v(1-v)\Delta\theta^2)^{3/2}}{a - \mathbb{E}[\theta]}\right). \qquad \text{(A.58)}$$

If $v < 1/2$, $\phi_p > 0$ may hold, i.e., entry can be insufficient. This can be seen by setting F at its lower bound [see ($P_2 + P_3$)], and choosing a choke price, a, which is sufficiently low.

If $v \geqslant 1/2$, $\phi_p < 0$ always holds, i.e., entry is excessive. To prove this claim, we can use (P_1) to get:

$$\frac{\sqrt{4bF - v(1-v)v\Delta\theta^2}}{a - \mathbb{E}[\theta]} < 1. \qquad \text{(A.59)}$$

Differentiating (A.58) with respect to F then implies:

$$\frac{\partial\phi_p}{\partial F} = \frac{1}{8}\left(-8 + 6\frac{\sqrt{4bF - v(1-v)v\Delta\theta^2}}{a - \mathbb{E}[\theta]}\right) < 0. \qquad \text{(A.60)}$$

Given (A.60), we can insert the lower bound for F [see ($P_2 + P_3$) and note $v > 1/3$] into

(A.58) to obtain:

$$sign[\phi_p] = sign[(a - \mathbb{E}[\theta])(1 - 3v) + v^2 \Delta\theta]. \tag{A.61}$$

Observing (A.6) shows that $a - \overline{\theta} > 0$ and, hence, $a - \mathbb{E}[\theta] > v\Delta\theta$. With $a - \mathbb{E}[\theta] = v\Delta\theta$, the sign of ϕ_p is equal to the sign of $1 - 3v + v$, which is equal to zero at $v = 1/2$ and negative for a higher v. Therefore, $\phi_p < 0$ holds for every $v \geqslant 1/2$.

Excessive and Insufficient Entry: Asymmetric Information

Using Lemma 2, we can rewrite (25) as:

$$\phi_a \equiv \left.\frac{\mathrm{d}\mathbb{E}[W_a(n)]}{\mathrm{d}n}\right|_{n=n_a^e} = \frac{R\left(\sqrt{Rb} + v\Delta\theta\right)}{a - \overline{\theta}} + \frac{v(3 - 4v)\Delta\theta^2}{8b(1 - v)} - F, \tag{A.62}$$

with $R = F - \frac{v}{(1-v)4b}\Delta\theta^2 > 0$.

If $v < \frac{1}{2}(2 - \sqrt{2})$, $\phi_a > 0$ may hold, i.e., entry can be insufficient. This can be seen by setting F at its lower bound [see ($A_2 + A_3$)], and by choosing a choke price, a, sufficiently low.

If $v \geqslant \frac{1}{2}(2 - \sqrt{2})$, $\phi_a < 0$ always holds, i.e., entry is excessive. To prove this statement, we first observe that $\phi_a > 0$ is more likely to hold the lower a. We can use (A_1) to compute the lower bound of a as:

$$\underline{a} = \overline{\theta} + \sqrt{4bF - \frac{v}{1 - v}\Delta\theta^2}. \tag{A.63}$$

Inserting (A.63) into (A.62) yields:

$$\phi_a(a = \underline{a}) = \frac{1}{4}\left(\frac{v\Delta\theta\left((1 - v)\sqrt{4bF - \frac{v\Delta\theta^2}{1-v}} + \Delta\theta(1 - 2v)\right)}{b(1 - v)} - 2F\right). \tag{A.64}$$

Differentiating (A.64) with respect to F and observing ($A_2 + A_3$) leads to:

$$\frac{\partial\phi_a(a = \underline{a})}{\partial F} = \frac{1}{4}\left(-2 + \frac{2v\Delta\theta}{\sqrt{4bF - \frac{v\Delta\theta^2}{1-v}}}\right) < 0. \tag{A.65}$$

Inserting the lower bound for F [see ($A_2 + A_3$) and note $v > \frac{1}{8}(7 - \sqrt{33})$] into (A.64), we observe that $\phi_a(a = \underline{a}) < 0$ if $v \geqslant \frac{1}{2}(2 - \sqrt{2})$. This completes the proof.

Welfare Comparison with Free Entry

In the following, we prove that $\mathbb{E}[W_a(n_a^e)] > \mathbb{E}[W_p(n_p^e)]$ can hold if $v < \overline{v} \approx 0.238036$ and if the choke price, a, is sufficiently large. The proof proceeds in three steps. First, we show that there is only one choke price for each informational structure, a_k, such that $n_k^e(a) > \overline{n}(a)\ \forall a > a_k$. Second, we define a unique choke price, $\hat{a}$, at which $n_a^e(\hat{a}) = n_p^e(\hat{a})$ holds and prove that $(\mathbb{E}[W_a(n_a^e)] - \mathbb{E}[W_p(n_p^e)])\Big|_{a=\hat{a}} < 0$. Third, we observe that the difference in expected welfare at the zero-profit conditions has a linear representation in the choke price, a, i.e., $\mathbb{E}[W_a(n_a^e)] - \mathbb{E}[W_p(n_p^e)] = a\Gamma(\cdot) + \Psi(\cdot)$ and we prove that $\Gamma(\cdot)$ can be positive if $v < \overline{v}$. Then, $\mathbb{E}[W_a(n_a^e)] - \mathbb{E}[W_p(n_p^e)] > 0$ is feasible if the choke price, a, is large enough.

Step 1.

Consider $\overline{n}$ and n_k^e as functions of the choke price. It is straightforward to see that $\overline{n}$ is strictly increasing and strictly concave in a, while n_a^e and n_p^e are linear in a. Hence, there are at most two intersections between $n_a^e(a)$, respectively $n_p^e(a)$, and $\overline{n}(a)$. Observing (11), and (19), it is clear that a relatively low choke price will imply that $n_k^e < 2$, while $\overline{n} > 2$ holds. In turn, a relatively high choke price will imply $n_k^e > \overline{n} > 2$. This shows that the difference $n_k^e(a) - \overline{n}(a)$ changes sign from negative to positive as the choke price increases. Therefore, the intersection points a_k are unique, and $n_k^e(a) > \overline{n}(a)$ holds $\forall a > a_k$.

Step 2.

Using (11) and (19), we can rewrite (21) and (22) as:

$$\mathbb{E}[W_p(n_p^e)] = \frac{b}{2}\left(\frac{(2(a-\mathbb{E}[\theta])-y)^2}{4b^2} + \left(\frac{2(a-\mathbb{E}[\theta])}{y} - 1\right)\mathrm{Var}[q_p(\theta)]\right), \tag{A.66}$$

$$\begin{aligned}\mathbb{E}[W_a(n_a^e)] = \frac{b}{2}&\left(\frac{(2(a-\overline{\theta})-x)^2}{4b^2} + \left(\frac{2(a-\overline{\theta})}{x} - 1\right)\mathrm{Var}[q_a(\theta)]\right)\\ &+ v\Delta\theta\frac{(2(a-\overline{\theta})-x)}{2b} - \left(\frac{2(a-\overline{\theta})}{x} - 1\right)\frac{(v\Delta\theta)^2}{2b(1-v)},\end{aligned} \tag{A.67}$$

respectively, where we used:

$$x \equiv \sqrt{4bF - \frac{v\Delta\theta^2}{1-v}}, \tag{A.68}$$

$$y \equiv \sqrt{4bF - v(1-v)\Delta\theta^2}, \tag{A.69}$$

to simplify notation. Using (A.68) and (A.69), we can write:

$$n_a^e = \frac{2(a-\overline{\theta})}{x} - 1, \tag{A.70}$$

$$n_p^e = \frac{2(a-\mathbb{E}[\theta])}{y} - 1. \tag{A.71}$$

Since $y > x$, it is straightforward to see that $\mathrm{d}n_a^e/\mathrm{d}a > \mathrm{d}n_p^e/\mathrm{d}a > 0$. Solving $n_a^e(\hat{a}) = n_p^e(\hat{a})$ with respect to $\hat{a}$ yields:

$$\hat{a} = \frac{v\Delta\theta x}{y-x} + \overline{\theta}. \tag{A.72}$$

Inserting (A.72) into (A.66) and (A.67), we obtain:

$$\left(\mathbb{E}[W_a(n_a^e)] - \mathbb{E}[W_p(n_p^e)]\right)\Big|_{a=\hat{a}} = -\frac{v\Delta\theta T\left((1-v)(y-x) + v^2\Delta\theta\right)}{4b(1-v)(y-x)} < 0, \tag{A.73}$$

with $T \equiv x + 2v\Delta\theta - y$. Note that $T > 0$ holds. This can be proven as follows. First, we have $\mathrm{d}x/\mathrm{d}F > \mathrm{d}y/\mathrm{d}F$ [see (A.68) and (A.69)], which implies $\mathrm{d}T/\mathrm{d}F > 0$. Second, we evaluate T at the lower bounds of F [see ($A_2 + A_3$)], and obtain:

$$T\left(F = \frac{v\Delta\theta^2}{4b(1-v)^2}\right) = \frac{v\Delta\theta}{1-v}\left(3 - 2v - \sqrt{3 - 3v + v^2}\right) > 0, \tag{A.74}$$

$$T\left(F = \frac{v(3-4v)\Delta\theta^2}{8b(1-v)}\right) = \frac{1}{2}\Delta\theta\left(4v + \sqrt{2}\left(\sqrt{\frac{v(1-4v)}{1-v}} - \sqrt{\frac{v\left(1-2v^2\right)}{1-v}}\right)\right) > 0. \tag{A.75}$$

Together with the first statement, this shows that T is always positive.

Next, we define $\hat{n} \equiv n_a^e(\hat{a}) = n_p^e(\hat{a})$. Given (A.73), we obtain $\hat{n} < \overline{n}$. However, we need $n_k^e > \overline{n}$ since otherwise $\bar{n}$ would not be in the feasible range of firms. As such, $a > a_k > \hat{a}$ has to hold because $\mathrm{d}n_k^e/\mathrm{d}a > 0$ and $n_k^e(a) > \overline{n}(a)\ \forall a > a_k$. In addition, we see that $\mathrm{d}n_a^e/\mathrm{d}a > \mathrm{d}n_p^e/\mathrm{d}a$, which implies that $n_a^e > n_p^e\ \forall a > a_k$.

Step 3.

We collect the term a in the difference $\mathbb{E}[W_a(n_a^e)] - \mathbb{E}[W_p(n_p^e)]$ to obtain a linear representation of the difference in expected welfare levels, which reads:

$$\mathbb{E}[W_a(n_a^e)] - \mathbb{E}[W_p(n_p^e)] = a\Gamma(F, v, \underline{\theta}, \overline{\theta}) + \Psi(F, v, \underline{\theta}, \overline{\theta}), \tag{A.76}$$

with

$$\Gamma(F, v, \underline{\theta}, \overline{\theta}) = \frac{(y-x)(-8bF(1-v) + 3v\Delta\theta^2 - 4v^2\Delta\theta^2) + v^2\Delta\theta^2(2-3v)x}{4b(1-v)xy} \tag{A.77}$$

and $\Psi(F, v, \underline{\theta}, \overline{\theta})$ a term which includes all the remaining parameters.

If $\Psi(\cdot) > 0$ holds, we obtain $\Gamma(\cdot) < 0$. This can be proven by setting $a = \hat{a}$ and observing that $\hat{a}\Gamma(\cdot) + \Psi(\cdot)$ is negative (see step 2). Moreover, $a > \hat{a}$ must hold, which implies that (A.76) will always be negative as long as $\Psi(\cdot) > 0$ holds.

If $\Psi(\cdot) < 0$ holds, a positive difference in expected welfare levels is possible. This is because $\Gamma(F, v, \underline{\theta}, \overline{\theta}) > 0$ can hold if $v < \overline{v}$, which leads to a positive sign of (A.76) for a sufficiently large choke price, a. Hence, we study whether $\Gamma(F, v, \underline{\theta}, \overline{\theta})$ can be positive under the constraint ($A_2 + A_3$). For $v < \frac{1}{8}(7 - \sqrt{33})$, the lower bound for F is $\frac{v(3-4v)\Delta\theta^2}{8b(1-v)}$. Using this expression in (A.77), and simplifying, we obtain:

$$\lim_{F \to \frac{v(3-4v)\Delta\theta^2}{8b(1-v)}} \Gamma(F, v, \underline{\theta}, \overline{\theta}) = \frac{v(2-3v)\Delta\theta\sqrt{\frac{v(1-2v^2)}{2(1-v)}}}{2b\left(1-2v^2\right)} > 0. \tag{A.78}$$

Therefore, by continuity, $\Gamma(\cdot)$ can be positive for every $v \in (0, \frac{1}{8}(7 - \sqrt{33})]$.

Conversely, suppose $v > \frac{1}{8}(7-\sqrt{33})$, and observe that numerator in (A.77) determines the sign of $\Gamma(\cdot)$. Therefore, we study under which conditions the following system admits solutions:

$$\begin{cases} (y-x)(-8bF(1-v) + 3v\Delta\theta^2 - 4v^2\Delta\theta^2) + v^2\Delta\theta^2(2-3v)x > 0 \\ F > \frac{v\Delta\theta^2}{4b(1-v)^2}. \end{cases} \tag{A.79}$$

We proceed with a "guess and verify" approach that there is a v large enough such that the system does not admit solutions. Hence, consider the largest v (say $\tilde{v}$) such that the second inequality in (A.79) almost binds, i.e., $\lim_{v\to\tilde{v}}(F - \frac{v\Delta\theta^2}{4b(1-v)^2}) = 0$. Substituting $F = \frac{\tilde{v}\Delta\theta^2}{4b(1-\tilde{v})^2}$ into the first inequality of (A.79) and rearranging yields:

$$\frac{\tilde{v}^2(3\tilde{v}^3 + (4\sqrt{\tilde{v}^2 - 3\tilde{v} + 3} - 9)\tilde{v}^2 + (9 - 7\sqrt{\tilde{v}^2 - 3\tilde{v} + 3})\tilde{v} + \sqrt{\tilde{v}^2 - 3\tilde{v} + 3} - 1)\Delta\theta^3}{(1-\tilde{v})^2}. \tag{A.80}$$

This term is positive if $\tilde{v} < \overline{v}$, with $\overline{v} \approx 0.238036$, and equal to zero if $v = \overline{v}$. Therefore, by continuity, $\Gamma(\cdot)$ can be positive for every $v \in (0, \overline{v})$. Since $\overline{v} < 2/3$, we also see that the condition on v is more restrictive than the one used in Proposition 1. Hence, if $v < \overline{v} \approx 0.238036$, we find that $\mathbb{E}[W_a(n_a^e)] > \mathbb{E}[W_p(n_p^e)]$ can hold if the choke price, a, is sufficiently large.

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